Princess Sumaya University for Technology
King Abdullah II School of Engineering

EE27355
Communication Principles

Quiz #3
Wednesday 25/3/2026

Name:……………………………………………………………… Section 2

Q.1)

(a) Find the Compact Fourier Series and sketch the amplitude and phase spectra for w(t) [see Figure Q.1], where the periodicity is equal 4. Please take the integration limits from -2 to 2. [14-Points]

(b) From the result that you got in part (a), find the exponential Fourier Series and sketch the amplitude and phase spectra for w(t). [6-Points]

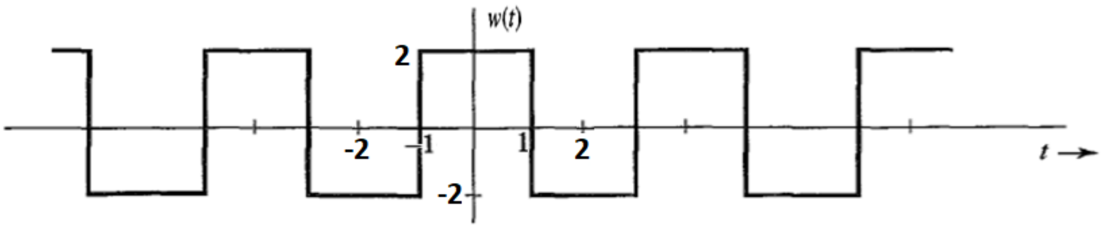


Figure Q.1

<u>Solution:</u> [20-Points]

$$a_0 = \frac{1}{T_0} \int_{T_0} g(t)\,dt$$

$$= \frac{1}{4} \int_{-2}^{2} g(t)\,dt$$

$$= \frac{2}{4} \left[\int_{-2}^{-1} -1\,dt + \int_{-1}^{1} 1\,dt + \int_{1}^{2} -1\,dt \right]$$

$$= \frac{2}{4} \left[-t\Big|_{-2}^{-1} + t\Big|_{-1}^{1} + -t\Big|_{1}^{2} \right]$$

$$= \frac{2}{4} \left[(+1-2) + (1+1) + (-2+1) \right]$$

$$= \frac{2}{4} \left[-1 + 2 - 1 \right]$$

$$= \emptyset$$

$$a_n = \frac{2}{T_0}\int_{T_0} g(t)\cos n\omega_0 t\,dt$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin ax + C$$

$$= \frac{2}{4}\int_{-2}^{2} g(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{4}(2)\int_{0}^{2} g(t)\cos n\omega_0 t\,dt$$

$$= 2\int_0^1 (1)\cos n\omega_0 t\,dt + 2\int_1^2 (-1)\cos n\omega_0 t\,dt$$

$$= \frac{2}{n\omega_0}\sin n\omega_0 t\Big|_0^1 - \frac{2}{n\omega_0}\sin n\omega_0 t\Big|_1^2$$

$$= \frac{4}{n\pi}\sin \frac{n\pi t}{2}\Big|_0^1 - \frac{4}{n\pi}\sin \frac{n\pi t}{2}\Big|_1^2$$

$$= \frac{4}{n\pi}\left[\sin\frac{n\pi}{2} - \varnothing\right] - \frac{4}{n\pi}\left[\cancelto{\varnothing}{\sin\frac{2n\pi}{2}} - \sin\frac{n\pi}{2}\right]$$

$$= \frac{8}{n\pi}\sin\frac{n\pi}{2}$$

$$a_n = \begin{cases} 0 & n \text{ even} \\ \frac{8}{n\pi} & n = 1, 5, 9, \dots \\ \frac{-8}{n\pi} & n = 3, 7, 11, \dots \end{cases}$$

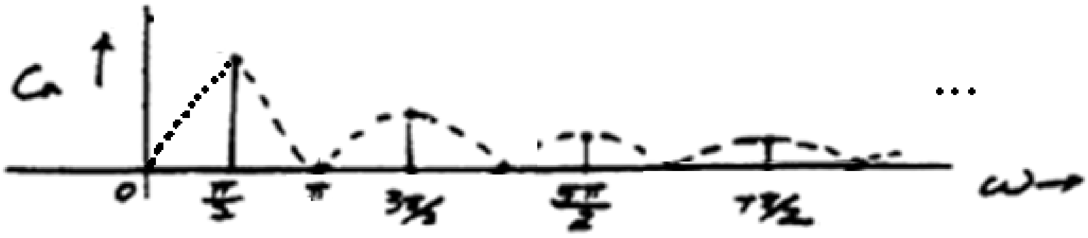
$$b_n = \varnothing$$

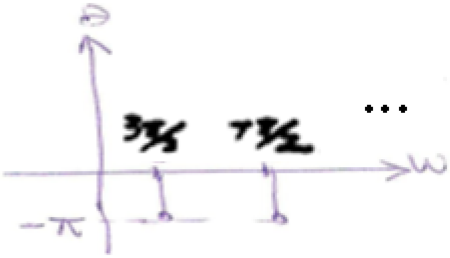
$$g(t) = \cancelto{\varnothing}{a_0} + \sum_{n=1}^{\infty}\left(a_n\cos n\omega_0 t + \cancelto{\varnothing}{b_n}\sin n\omega_0 t\right)$$

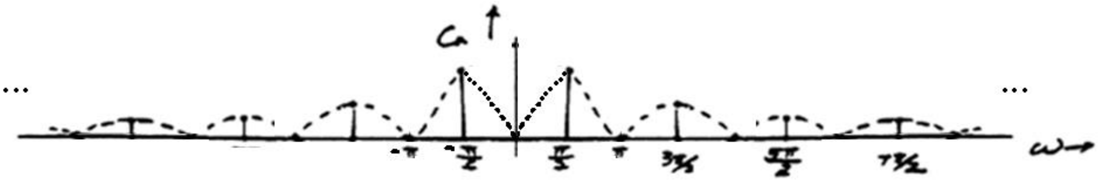
$$= \frac{8}{\pi}\left(\cos\frac{\pi t}{2} - \frac{1}{3}\cos\frac{3\pi}{2}t + \frac{1}{5}\cos\frac{5\pi}{2}t - \frac{1}{7}\cos\frac{7\pi}{2}t + \frac{1}{9}\cos\frac{9\pi}{2}t + \dots\right)$$

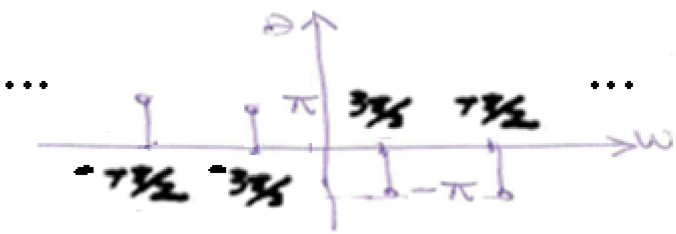
$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n\cos(n\omega_0 t + \theta_n) \qquad -\cos x = \cos(x-\pi)$$

$$= \frac{8}{\pi}\left(\cos\frac{\pi}{2}t + \frac{1}{3}\cos\left(\frac{3\pi}{2}t - \pi\right) + \frac{1}{5}\cos\frac{5\pi}{2}t + \frac{1}{7}\cos\left(\frac{7\pi}{2}t - \pi\right) + \frac{1}{9}\cos\frac{9\pi}{2}t + \dots\right)$$









Hint:

Trignometric Fourier Series:

$$a_0 = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} g(t)\,dt$$

$$a_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} g(t)\cos n\omega_0 t\,dt \qquad n = 1, 2, 3, \ldots$$

$$w(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$b_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} g(t)\sin n\omega_0 t\,dt \qquad n = 1, 2, 3, \ldots$$

Compact Fourier Series:

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n) \qquad t_1 \le t \le t_1 + T_0$$

$$C_0 = a_0 \qquad C_n = \sqrt{a_n^2 + b_n^2} \qquad \theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

Exponential Fourier Series:

$$g(t) = D_0 + \sum_{n=1}^{\infty} D_n e^{jn\omega_0 t} + D_{-n} e^{-jn\omega_0 t} = D_0 + \sum_{n=-\infty\ (n\neq 0)}^{\infty} D_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$D_0 = a_0 = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} g(t)\,dt \qquad D_n = \frac{1}{T_0}\int_{T_0} g(t)e^{-jn\omega_0 t}\,dt$$

$$\int e^{cx}\,dx = \frac{1}{c}e^{cx} \qquad \int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c \qquad \int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

$$\sin x = \mathrm{Im}\{e^{ix}\} = \frac{e^{ix} - e^{-ix}}{2i} \qquad \cos x = \mathrm{Re}\{e^{ix}\} = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sin^2(t) + \cos^2(t) = 1 \qquad \sin(-t) = -\sin(t) \qquad \cos(-t) = \cos(t)$$

$$\sin^2(x) = \tfrac{1}{2}[1 - \cos(2x)] \qquad \cos^2(x) = \tfrac{1}{2}[1 + \cos(2x)]$$

$$\sin(x) = \cos(90° - x) \qquad \cos(x) = \sin(90° - x)$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$